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APPENDIX A

Proof of (39) to (42)

The selectivity S is defined as

$$S(t) = \frac{\overline{B}(t)}{\overline{C}(t)} = \frac{\sum_{i=1}^{N} B_i(t)}{\sum_{i=1}^{N} C_i(t)} \tag{A1}$$

Differentiation of (A1) yields

$$\overline{C}^2 \frac{dS}{dt} = \sum_{i=1}^{N} C_i \sum_{i=1}^{N} \frac{dB_i}{dt} - \sum_{i=1}^{N} B_i \sum_{i=1}^{N} \frac{dC_i}{dt} \tag{A2}$$

Now

$$C_i \frac{dB_i}{dt} = B_i \frac{dC_i}{dt} = \frac{k_i k_i^* A_i^2(0)}{k_i + k_i^*} X_i(1 - X_i) \tag{A3}$$

where

$$X_i = \exp[-(k_i + k_i^*)t] \tag{A4}$$

Use of (A3) and simple algebraic manipulations enable transformation of (A3) into

$$\overline{C}^2 \frac{dS}{dt} = \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{(k_i k_j^* - k_i^* k_j) A_i(0) A_j(0)}{(k_i + k_i^*)(k_j + k_j^*)} F_{ij} \tag{A5}$$

where

$$F_{ij} = (k_i + k_i^*)X_i + (k_j + k_j^* - k_i - k_i^*)X_i X_j - (k_j + k_j^*)X_j \tag{A6}$$

Note that we order the species such that

$$k_i + k_i^* \leqq k_j + k_j^* \quad \text{for} \quad j > i \tag{A7}$$

and therefore

$$k_j + k_j^* - k_i - k_i^* \geqq 0 \tag{A8}$$

According to Hardy et al. (1934, p. 74), any convex function ψ satisfies the following inequality:

$$\frac{\sum_{i=1}^{N} p_i \psi(y_i)}{\sum_{i=1}^{N} p_i} \geqq \psi \left(\frac{\sum_{i=1}^{N} p_i y_i}{\sum_{i=1}^{N} p_i} \right) \tag{A9}$$

where p is a non-negative function. Substitution of

$$\begin{aligned} p_1 &= k_i + k_i^* \quad p_2 = k_j + k_j^* - k_i - k_i^* \\ y_1 &= k_i + k_i^* \quad y_2 = k_j + k_j^* + k_i + k_i^* \\ \psi(y) &= e^{-yt} \end{aligned} \tag{A10}$$

into (A9) yields

$$\frac{(k_i + k_i^*)X_i + (k_j + k_j^* - k_i - k_i^*)X_j X_i}{k_j + k_j^*} \geqq \exp\left[\frac{-(p_1 y_1 + p_2 y_2)t}{k_j + k_j^*}\right] = X_j \tag{A11}$$

(A11) implies that

$$F_{ij} \geqq 0 \quad \text{for} \quad j > i \tag{A12}$$

Hence, (A5) predicts that (39) implies (40), while (41) guarantees that (42) is valid.

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An Analogy for Heat Transfer with Wavy / Stratified Gas-Liquid Flow

The von Karman analogy between heat transfer and momentum transfer in turbulent fluids is shown to apply to heat transfer through wavy liquid films in horizontal, stratified, gas-liquid flow. Nusselt numbers predicted from the analogy are shown to be in good agreement with experimental data for air-water flows involving three-dimensional wavy films and large-amplitude roll waves. The Nusselt numbers encountered with these turbulent water films are found to be in the same range as those associated with condensation of organics in horizontal tubes under stratified flow conditions.

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SCOPE

Stratified and annular two-phase, gas-liquid flows occur in a variety of process equipment ranging from cooler condensers, thermosyphon reboilers, and nuclear reactor cooling channels to thin liquid film gas-liquid reactors and gas-absorption columns. In sulfonation reactors and other highly exothermic reaction systems involving gas-liquid reactions, thin liquid films are employed to reduce the resistance to heat transfer to the reactor wall from the reacting species. At the relatively high flow rates encountered in many process systems, the liquid film is wavy and turbulent, and any hydrodynamic analyses are complicated by the variety of wave structures that exist.

Chu and Dukler (1974) and Dukler (1972) have studied and statistically analyzed such wave structure for falling liquid films, and Cohen and Hanratty (1965, 1968), Hanratty and Engen (1957), Hanratty and Hershman (1961), Davis (1969), Narasimhan and Davis (1972), and Miya, Woodmansee, and Hanratty (1971) have studied wave characteristics for horizontal, stratified, gas-liquid flow. Frisk and Davis (1972) measured the effects of wave action on heat transfer to stratified gas-liquid flow, finding that waves enhance heat transfer between the liquid film and a solid wall by about 100%. These studies suggest that for both annular and stratified flows involving wavy films, the main resistance to the transfer of heat and mass through the liquid film is a thin liquid substrate. Large amplitude waves (roll waves) and small amplitude waves (capillary waves at the gas-liquid interface) appear to move over this liquid substrate without inducing additional turbulence in the substrate.

The success of the analogies of Dukler (1959) and Hewitt (1961) in predicting heat transfer characteristics of upward and downward annular gas-liquid flows and the reasonably successful application of the von Karman analogy by Rosson and Myers (1965) to predict condensing coefficients for condensation in a horizontal pipe indicate that analogies between heat transfer and momentum transfer are quite suitable for relatively nonwavy flows.

It is the purpose of this study to show that the von Karman analogy can also be applied with considerable success to stratified gas-liquid flows involving large and small amplitude waves. Furthermore, the analogy is written in a form that can be readily used by design engineers for making heat transfer predictions.

CONCLUSIONS AND SIGNIFICANCE

The effect of waves in stratified gas-liquid flow is to increase the heat transfer to wavy liquid films compared with smooth film flow. The increase in wall shear stress associated with wave action accompanies an increase in heat transfer, and this relationship between increased shear stress and increased heat transport suggests that conventional analogies might apply to these hydrodynamically complicated flows. In this study we show that the von Karman analogy between heat transfer and momentum transfer can be successfully applied to predict the heat transfer characteristics of three-dimensional waves and roll waves encountered with stratified gas-liquid flows.

The von Karman analogy as applied by Rosson and Myers (1965) to predict heat transfer for stratified flow in condensers is in good agreement with new data on heat transfer to horizontal wavy/stratified air-water flow conditions corresponding to three-dimensional waves and large amplitude roll waves. The data can also be correlated with a single Reynolds number based on liquid phase properties, which suggests that the primary effect of an increase in the gas flow rate is to decrease the film thickness and increase the shear stress and heat transfer rate in the liquid film. The gas flow rate enters only implicitly through the average film thickness.

The agreement between analysis and experiment suggests that the turbulence characteristics in the viscous sublayer and buffer region of a wavy film flow are not greatly different than for a single-phase flow. The most important parameter that must be known to predict heat transfer characteristics is the wall shear stress. Although the use of an average wall shear stress appears to be quite accurate for predicting the heat transfer characteristics of three-dimensional wavy film flow, the intermittent nature of large-amplitude roll waves leads to more scatter in the prediction of heat transfer characteristics from average wall shear stresses and film thicknesses.

When a gas flows concurrently with a liquid film in a conduit or channel, the gas-liquid interface remains smooth and stable only at low flow rates of each phase. As the gas flow rate is increased, two-dimensional ripples and then three-dimensional small amplitude waves form at the liquid surface. At yet higher gas flow rates, large-amplitude roll waves produce an unsteady, surging motion of the liquid film, and at even higher flow rates droplets break off the crests of the roll waves and dispersed/stratified flow occurs.

Frisk and Davis (1972) showed that two-dimensional waves or surface ripples do not enhance heat transfer between the liquid film and the wall compared with smooth laminar film flow. But three-dimensional waves and roll waves significantly affect such heat transfer. Furthermore, they found that three-dimensional waves and roll waves produce very nearly the same enhancement despite the apparently different hydrodynamics. These results suggest that the predominant resistance to heat transfer in wavy film systems is the thin base film over which waves move. Dukler (1972) has made the same speculation based on his studies of the wave structure in falling liquid films.

This paper represents an extension of the work of Frisk and Davis to predict the heat transfer characteristics of wavy/stratified gas-liquid flows.

EXPERIMENTS

New experiments have been carried out in the water/wind tunnel facility described by Frisk and Davis (1972) and Narasimhan and Davis (1972), and only a brief recapitulation of the experimental system is needed here. Air was delivered to a rectangular Plexiglas test section (25 cm wide and 2.5 cm high) from a turbocompressor. The air was humidified and metered through a venturi before coming into contact with the water stream, which was introduced from a constant head tank through perforations in the bottom of the tunnel. Approximately 7 m from the air and water inlets the Plexiglas bottom was replaced by a 60-cm-long copper block which served as the heat transfer test section. The thick copper block was heated electrically by means of a bank of powerstats in such a way that

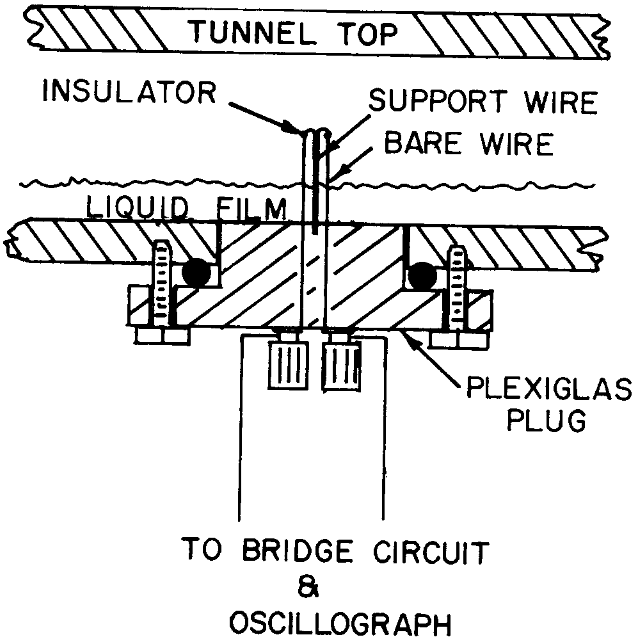

Fig. 1. The conductance probe for film thickness measurement.

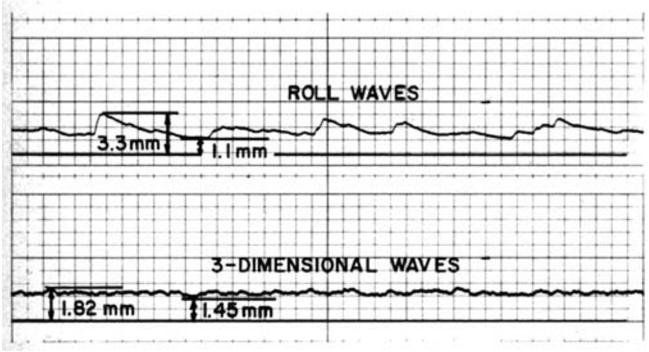

Fig. 2. Oscillograph tracings of wave shapes.

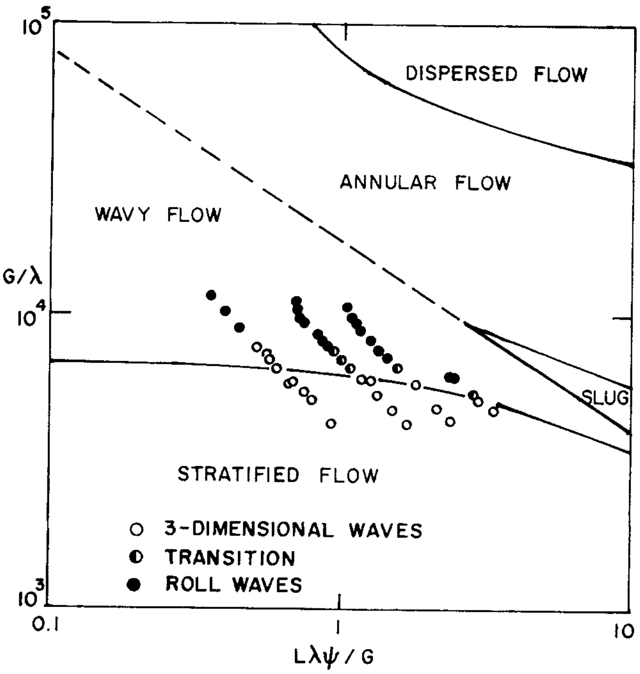

Fig. 3. The modified Baker chart with the range of experimental data.

either constant wall temperature or constant wall heat flux boundary conditions could be applied. The wall temperature profile and thermal characteristics were measured as discussed by Frisk and Davis (1972).

It should be noted here that heat balances were made by measuring the mixing cup temperature rise of the water and comparing the enthalpy increase of the water with the electrical power supplied to the heat transfer test section. As the air was saturated at the inlet to the water/wind tunnel, and as the mixing cup temperature rise of the water was kept small ($\Delta T < 5°C$), the heat transfer to the gas phase was generally negligible. At high air flow rates, however, the heat balances were not sufficiently accurate to place confidence in the heat transfer data, so these results are not included here. Although data were taken up to the onset of dispersed flow, only the data up to a gas phase Reynolds number Re_G of 22,000 are presented. Data were rejected if the heat balance was not accurate to within 5%.

One change in the experimental techniques was to replace the platinum electrodes used in the previous work by a two-wire conductance probe shown in Figure 1. By measuring the electric current conducted through the liquid between the wires, the film thickness was obtained. This conductance probe was calibrated by use of the micrometer probe discussed by Narasimhan and Davis (1972). Typical film thickness profiles obtained with the two-wire probe are shown in the oscillograph tracings of Figure 2. For the three-dimensional waves, typical peak-to-peak distances are 1 cm, and for roll waves the distance between wave crests is about 30 cm. Since the roll waves do not all travel at the same velocity, the latter distance is merely a crude approximation to an average spacing.

Figure 2 shows clearly that the wave characteristics are significantly different for the two types of waves. The three-dimensional waves appear to be surface waves that do not penetrate deeply into the liquid film, but the roll waves appear to disturb all but a rather thin layer of fluid. We might be led to expect different heat transfer characteristics for the two wave types, but as we shall show, this is not the case.

A rectangular test section with large aspect ratio has been used to approximate a two-dimensional flow system. With this system more precise measurements of the fluid mechanics parameters (film thickness, wave characteristics, shear stress, etc.) can be made than with a tube of circular cross section. Since it is the purpose of the study to examine the heat transfer characteristics of wavy liquid films, the results are applicable to other geometries provided that the wave structure is similar. As we shall show, the results of Rosson and Myers (1965) on heat transfer in horizontal condenser tubes do not differ greatly from the results of our work.

HEAT TRANSFER DATA

Heat transfer data were taken over a range of air and water flow rates from the onset of three-dimensional waves to the onset of dispersed/stratified flow. Although the Baker (1954) chart and the modified Baker chart of Scott (1963) are based on circular tube data, the transition from stratified flow to wavy flow of the modified Baker chart corresponds fairly closely to the transition from small-amplitude three-dimensional waves to large-amplitude roll waves in our system, as shown in Figure 3. As it is convenient and informative to show the flow conditions of this study on a modified Baker chart, this is done in Figure 3. Only the data for which acceptable heat balances were obtained are shown.

Defining the characteristic length of the liquid film as its average thickness $\bar{\delta}$, we may write the Nusselt number as $Nu^* = h\bar{\delta}/k_L$, where the heat transfer coefficient h is defined in terms of the local heat flux at the wall q_w and the temperature difference between the wall and the liquid mixing-cup temperature $T_w - \bar{T}_L$; that is,

$$h = q_w/(T_w - \bar{T}_L) \tag{1}$$

The average film thickness was calculated from the oscillograph tracings of the wave thickness by averaging over

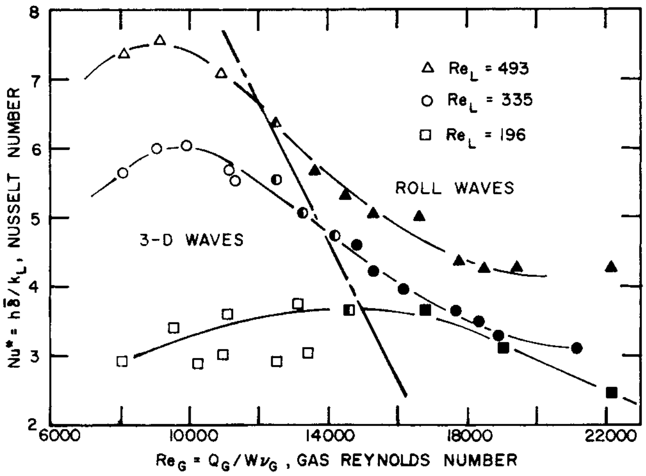

Fig. 4. Nusselt numbers as a function of gas and liquid Reynolds numbers.

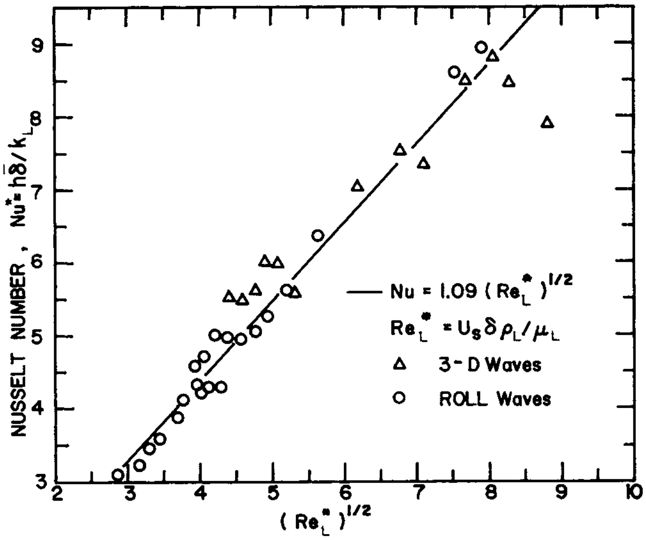

Fig. 5. Empirical correlation of the heat transfer data.

several waves; the heat flux and wall temperature were measured as indicated by Frisk and Davis (1972) and Davis and Cooper (1969), and the mixing-cup temperature was calculated by heat balance.

It is convenient at this point to define the gas and liquid Reynolds numbers in terms of the volumetric flow rate per unit width of tunnel for each phase; that is,

$$Re_L = Q_L/W\nu_L \tag{2}$$

and

$$Re_G = Q_G/W\nu_G \tag{3}$$

Typical heat transfer results are shown in Figure 4 for various values of Re_L and Re_G. Because the average film thickness decreases with an increase in Re_G, Figure 4 is somewhat misleading. The heat transfer coefficient actually increases with an increase in Re_G, but the product $h\bar{\delta}$ tends to decrease for a fully turbulent gas flow ($Re_G >$ 10000). Figure 4 shows that the data for three-dimensional waves and roll waves lie on smooth curves with no abrupt changes in Nusselt number associated with the transition from three-dimensional waves to roll waves.

Although Figure 4 indicates the effects of the various parameters (Re_L, Re_G and the wave structure) on the Nusselt number, the data for all of the experiments are well correlated in terms of a single Reynolds number, defined by

$$Re_L^* = U_{L,S}\bar{\delta}/\nu_L \tag{4}$$

where $U_{L,S}$ is the superficial liquid velocity based on the entire cross section of the channel. Figure 5 shows all of the data for three-dimensional wave flow and roll wave flow plotted as Nu^* vs. $\sqrt{Re_L^*}$. The data are in good agreement with the equation

$$Nu^* = 1.09\sqrt{Re_L^*} \tag{5}$$

There are no significant differences between the data for three-dimensional waves and roll waves, but the roll wave data show somewhat more scatter, probably due to the difficulty of defining and calculating a unique and meaningful average film thickness for the large amplitude roll waves.

It is significant that the heat transfer data can be correlated by means of a single Reynolds number Re_L^* based on liquid phase properties. This somewhat surprising result suggests that the only significant effect of the gas flow rate on the heat transfer through the liquid film is due to its effects on the liquid film thickness and the shear stress. The primary effect of an increase in the gas flow rate is to decrease the average film thickness and increase the wall shear stress. The turbulence characteristics of the gas phase have rather little effect on the transport of heat through the liquid film. The use of the superficial liquid velocity $U_{L,S}$ in the definition of Re_L^* decouples the gas and liquid velocities and incorporates only the effect of liquid flow rate. Any other liquid velocity of the system (for example, the average velocity of the liquid or the interfacial velocity) is dependent on the gas velocity, and so the definition of Re_L^*, Equation (4), involves the gas flow implicitly in $\bar{\delta}$ only.

Equation (5) is a good fit of the experimental data, but because it involves the average film thickness, it is not convenient for design purposes. The form of Equation (5) suggests an alternate approach to the analysis and correlation of data, for it has the same square-root dependence on superficial liquid velocity as the analysis of Rosson and Myers (1965), who applied the von Karman analogy to predict heat transfer coefficients for horizontal condensers.

THE VON KARMAN ANALOGY

By applying the von Karman analogy together with the Lockhart-Martinelli (1949) method for estimating the wall shear stress, Rosson and Myers obtained

$$Nu_{RM} \equiv \frac{hD}{k_L} = \frac{\phi\sqrt{8Re_{RM}}}{\left[5 + \frac{5}{Pr}\ln(5Pr + 1)\right]} \tag{6}$$

where the Lockhart-Martinelli parameter ϕ is defined by

$$\phi^2 = \frac{(\Delta P/L)_{TP}}{(\Delta P/L)_L} \tag{7}$$

and the Reynolds number Re_{RM} is defined in terms of the tube diameter D and the superficial velocity of the liquid; that is

$$Re_{RM} = DU_{L,S}/\nu_L \tag{8}$$

The similarity between Equations (5) and (6) suggests that the von Karman analogy can be applied to the wavy flows considered here. As the analysis parallels that of Rosson and Myers, we shall merely outline the analysis here.

For a turbulent flow field, the ratio of the heat flux q to the momentum flux τ is given by

$$\frac{q}{\tau} = C_p \left[\frac{(\nu/Pr) + (\epsilon_M/Pr_t)}{\nu + \epsilon_M} \right] \frac{dT}{du} \tag{9}$$

where the turbulent Prandtl number is given by $Pr_t = \epsilon_M/\epsilon_H$. For the relatively simple parallel-flat-plate flow system considered here, the shear stress τ and the heat flux are essentially independent of the distance from the heat surface and may be taken to be their values at the wall; that is, $\tau = \tau_w$ and $k = q_w$.

Note that for a smooth film flowing over a flat plate under the influence of a constant interfacial shear, a force balance on the liquid film gives

$$\tau_w = \tau + \frac{\Delta P}{L} y \tag{10}$$

where y is the distance from the solid boundary. For the interface $y = \delta$, Equation (10) gives

$$\tau_w = \tau_i + \frac{\Delta P}{L} \delta \tag{11}$$

Thus, if the liquid film is sufficiently thin and/or if the pressure drop $\Delta P/L$ is sufficiently small, the shear stress is independent of y and is given by $\tau = \tau_w = \tau_i$. We shall assume that this same result applies to the wavy film system.

Now, assuming that q and τ are constant and that the von Karman universal velocity profile applies to the liquid film over the laminar sublayer and the buffer layer, we may integrate Equation (9) to give the temperature difference across these layers; that is

$$\Delta T = \frac{5Pr}{C_p} \frac{q_w}{\rho_L} \sqrt{\frac{\rho_L}{\tau_w}} \left[1 + \frac{Pr_t}{Pr} \ln\left(1 + 5\frac{Pr}{Pr_t}\right) \right] \tag{12}$$

which is the result obtained by Rosson and Myers where they set $Pr_t = 1$.

Defining a heat transfer coefficient by $h = q_w/\Delta T$, and introducing the equivalent diameter $D_{eq} = 4WH/2(W + H)$ for a rectangular duct of height H and width W, we may write the Nusselt number as

$$Nu \equiv \frac{hD_{eq}}{k_L} = \frac{D_{eq}}{5\nu_L} \frac{\sqrt{\tau_w/\rho_L}}{\left[1 + \frac{Pr_t}{Pr} \ln\left(1 + 5\frac{Pr}{Pr_t}\right) \right]} \tag{13}$$

Equation (13) does not involve the film thickness explicitly, but it requires knowledge of the wall shear stress τ_w.

Although wall shear measurements were not made for most of the experiments of the present work, several investigators have reported interfacial shear measurements for the range of flow conditions of this study. Hanratty and Engen (1957), Ellis and Gay (1959), Smith and Tait (1966), Davis (1969), and Miya, Woodmansee, and Hanratty (1971) reported such results. The data of Hanratty and Engen, of Davis, and of Miya, et al., are particularly relevant. Davis (1969) compared the results of the earlier measurements, finding agreement between the data of Hanratty and Engen and Davis. They reported values of the interfacial friction factor $f_i \equiv \tau_i/\rho_G\overline{U}_G^2$ for $10930 \leq (H - \overline{\delta})\ \overline{U}_G/\nu_G \leq 41100$ and $65 \leq Re_L \leq 508$, but they did not attempt to correlate the data. Over this range of flow conditions, $0.00291 \leq f_i \leq 0.0053$. Although the scatter is appreciable, f_i appears to be approximately independent of Re_G for a given Re_L for $Re_L < 500$. Miya et al., reported interfacial friction factor data for the three-dimensional wave experiments of Cohen and Hanratty (1968) and their own results for roll waves. Defining an interfacial friction factor by $\tau_i = f_s/2\ \rho_G(\overline{U}_G - c)^2$, they showed that f_s is a linear function of Re_L for roll waves in the range $200 < Re_L < 900$ and for three-dimensional waves in the range $100 < Re_L < 1700$. If $\overline{U}_G >> c$, the interfacial friction factor f_i defined above is related to f_s by $f_i = f_s/2$. We can, therefore, use the available data for f_i to test the validity of Equation (13).

Thus, if we assume that $\tau_w = \tau_i$ from Equation (11) and if we introduce f_i, Equation (13) becomes

$$Nu = \frac{D_{eq}\overline{U}_G}{5\nu_L} \frac{\sqrt{f_i\,\rho_G/\rho_L}}{\left[1 + \frac{Pr_t}{Pr} \ln\left(1 + 5\frac{Pr}{Pr_t}\right) \right]} \tag{14}$$

It is convenient to write Equation (14) in an alternate form by writing the volumetric flow rate of the gas as $Q_G = (H - \overline{\delta})\ W\overline{U}_G$ and by introducing the gas Reynolds number $Re_G = Q_G/W\nu_G = (H - \overline{\delta})\overline{U}_G/\nu_G$. Eliminating $\overline{U}_G$ from Equation (14), we obtain

$$Nu = \frac{D_{eq}}{(H - \delta)} \frac{\mu_G}{\mu_L} \frac{Re_G \sqrt{f_i\,\rho_L/\rho_G}}{5\left[1 + \frac{Pr_t}{Pr} \ln\left(1 + 5\frac{Pr}{Pr_t}\right) \right]} \tag{15}$$

Equation (15) indicates that if f_i is constant, the product $(1 - \overline{\delta}/H)Nu$ is directly proportional to Re_G for fixed values of Pr_t and of the physical properties. This result can be compared with our data if we can assign values to Pr_t.

Several single-phase-flow heat transfer studies have been carried out to evaluate the turbulent Prandtl number Pr_t, and it has generally been found that $Pr_t < 1$. For a circular tube, Azer and Chao (1960) proposed the following correlation for $0.6 < Pr < 15$:

$$Pr_t = \frac{1 + 57Re^{-0.46}\,Pr^{-0.58} \exp\left[-\,(y/y_o)^{0.25}\right]}{1 + 135Re^{-0.45} \exp\left[-\,(y/y_o)^{0.25}\right]} \tag{16}$$

where y is the distance from the solid wall, and y_o is the tube radius. Hatton, Quarmby, and Grundy (1964) applied Azer and Chao's correlation to heat transfer with turbulent flow between parallel plates, and Larson and Yerazunis (1973) reported that their results for mass transfer in turbulent flow were consistent with values of Pr_t obtained by Azer and Chao.

Although we cannot expect Equation (16) to apply rigorously here, it is useful to determine the range of Pr_t predicted for single-phase flow from Equation (16) for the Prandtl number encountered here ($Pr = 5.85$). For $Re = 10^4$ (fully developed turbulent flow in a circular tube) and $Pr = 5.85$, Equation (16) gives $0.413 \leq Pr_t \leq 0.620$, and for 10^6 and $Pr = 5.85$ the equation gives $0.816 \leq Pr_t \leq 0.922$. We note that Pr_t can be as low as 0.413 for fully turbulent flow in a tube. Without further information about Pr_t for the turbulent film flows of interest, we can say little about the value of Pr_t, but we might anticipate that Pr_t is no lower than for tube flows. We shall assume that $Pr_t = 1$ for wavy film systems.

COMPARISONS WITH THE VON KARMAN ANALOGY

Using the friction factor data of Cohen and Hanratty and Miya, Woodmansee and Hanratty, and choosing $Pr_t = 1$, we obtain the comparisons between Equation (15) and the experimental results shown in Figures 6 and 7.

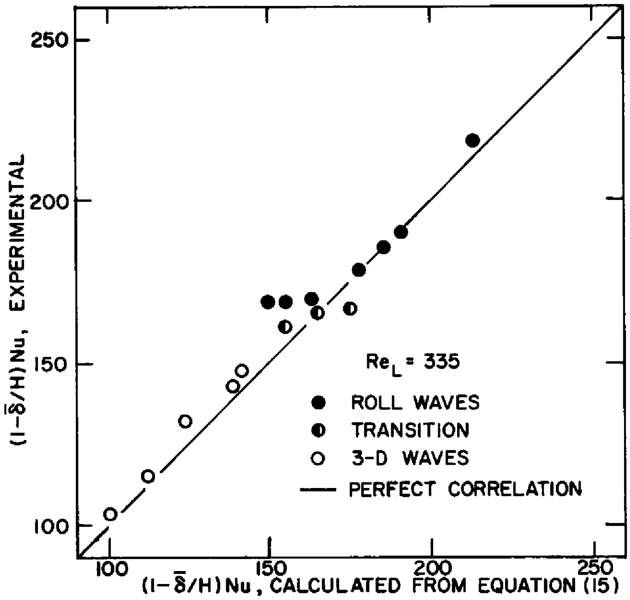

Fig. 6. Comparison of the data for Re_L = 335 with equation (15) with f_i = 0.0075 for 3-D waves and F_i = 0.0055 for roll waves.

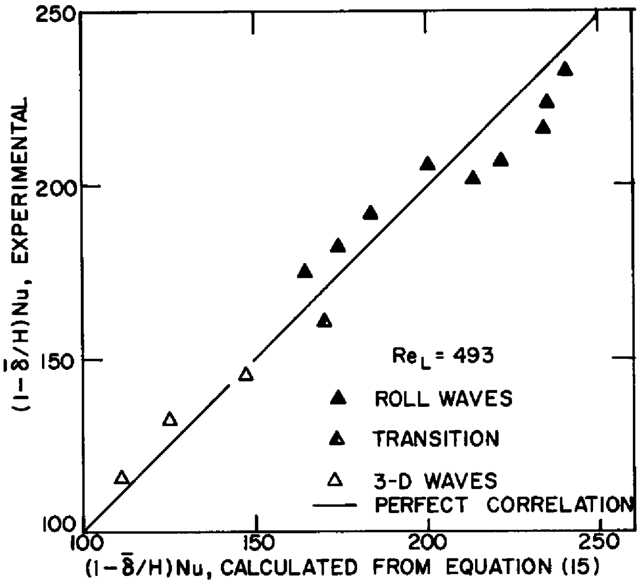

Fig. 7. Comparison of the data for Re_L = 493 with equation (15) with f_i = 0.0090 for 3-D waves and f_i = 0.0070 for roll waves.

The results for three-dimensional waves are in good agreement with theoretical predictions, and, although there is more scatter in the data for roll waves, the roll wave data also agree with the von Karman analogy based on friction factor data.

It is not surprising that the roll wave data show more scatter than three-dimensional wave data because of the unsteady nature of roll waves. The fluctuations in wall shear stress and film thickness are so great with roll waves that it is not entirely reasonable to use average values of film thickness and wall shear stress to predict heat transfer characteristics. Miya, et al. (1971), showed that the film height and shear stress profiles for roll waves are very similar, the maximum in the shear stress corresponding to the wave crest.

It is not possible to provide a more stringent test of the von Karman analogy or other analogies between heat and momentum transfer without detailed knowledge of the wall shear stress, and to obtain more detailed information we have performed preliminary measurements of wall shear using a hot film probe installed flush with the tunnel bottom just upstream of the heat transfer test section. Figure 8 shows oscillograph tracings of wall shear and film thickness for typical three-dimensional wave and roll wave conditions. For three-dimensional waves, the more regular fluctuations of the signals around a mean value make it much easier to define average values of wall shear and film thickness for these small-amplitude waves than it is for roll waves. The signals from the hot film probe shown in Figure 8 indicate the turbulent nature of the wavy film flows encountered here. As shown by Miya, et al., and by our data, roll waves are accompanied by large increases in the wall shear stress. The heat transfer data indicate that this increase in wall shear stress is in turn accompanied by increased heat transfer rates compared with the smooth film data of Frisk and Davis.

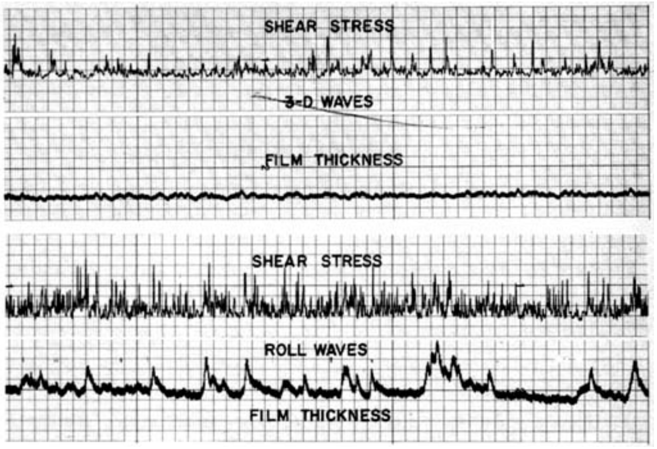

Fig. 8. Oscillograph tracings of wall shear stresses and film thicknesses.

COMMENTS

The reasonably good agreement between the experimental data for wavy/stratified air-water flow and the von Karman analogy suggests that the wave structure does not appreciably affect the laminar sublayer and the buffer layer of the liquid film compared with single-phase flow for the same τ_w. The turbulent flow in the region farthest from the wall is undoubtedly affected by the waves and cannot be expected to be similar to single-phase turbulent flow because of the interaction between the two phases. The convective motion associated with the waves appears to produce such mixing that the total resistance to heat transfer is in the laminar sublayer and the buffer layer. The wave action and interfacial roughness increase the interfacial shear and the wall shear compared with smooth film flow, and the result is the increase in heat transfer reported by Frisk and Davis (1972) and examined in more detail here.

Although the scatter in the present data is considerably less than that of Rosson and Myers, the Nusselt numbers encountered with the wavy/stratified flows of this study are in the same range as those reported by Rosson and Myers for methanol and acetone condensation in a horizontal pipe with stratified flow. Most of their theoretical and experimental results are in the range of $40 \leq Nu \leq 250$, where Nu is based on the equivalent diameter of the system. The present study represents a more rigorous test of the von Karman analysis because it is easier to control and measure the experimental parameters in the flow configuration used here than for other geometries.

It should be pointed out that the effect of the Prandtl number on the Nusselt number predicted by the von Karman analogy has not been adequately tested because the range of Prandtl numbers covered in this study and by Rosson and Myers is small. For very large Prandtl numbers the analogy under consideration is probably not applicable because turbulence in the viscous sublayer becomes important then.

The results obtained here for wavy/stratified flow lend support to the application of heat transfer and momentum transfer analogies to other two-phase flow configurations, particularly annular flow with a wavy interface. The analogies proposed by Dukler (1959) and Hewitt (1961) for vertical annular flows with a smooth gas-liquid interface should apply to wavy/annular flows with reasonably good accuracy, provided that accurate estimates of the wall shear stress can be made.

NOTATION

c = wave velocity, m/s
C_p = heat capacity of the liquid, J/kg°K
D = tube diameter, m
$D_{eq} = 4HW/2(H + W)$ = equivalent diameter, m
$f_i = \tau_i/\rho_G \overline{U}_G^2$ = interfacial friction factor, dimensionless
f = friction factor, dimensionless
f_s = friction factor defined by Miya et al. (1971), dimensionless
h = heat transfer coefficient, J/m^2 s°K
H = tunnel height, m
k = thermal conductivity, J/m s°K
$Nu = hD_{eq}/k_L$ = Nusselt number, dimensionless
$Nu^* = h\overline{\delta}/k_L$ = Nusselt number, dimensionless
$\Delta P/L$ = pressure drop per unit length, N/m^2
$Pr = C_p\mu/k$ = Prandtl number, dimensionless
$Pr_t = \epsilon_M/\epsilon_H$ = turbulent Prandtl number, dimensionless
q = heat flux, J/m^2 s
Q = volumetric flow rate, m^3/s
$Re = Q/W\nu$ = Reynolds number, dimensionless
$Re_L^* = \overline{\delta}U_{L,S}/\nu_L$ = Reynolds number, dimensionless
$Re_{RM} = DU_{L,S}/\nu_L$ = Reynolds number defined by Rosson and Meyers, dimensionless
$Re_{L,S} = D_{eq}U_{L,S}/\nu_L$ = Reynolds number, dimensionless
T = temperature, K
ΔT = temperature difference, °K
u = velocity, m/s
U = velocity, m/s
$\overline{U}_G$ = average velocity of the gas, m/s
$U_{L,S}$ = superficial velocity of the liquid, m/s
W = tunnel width, m
y = distance from solid boundary, m

Greek Letters

δ = film thickness, m
$\overline{\delta}$ = average film thickness, m
ϵ_H = eddy diffusivity for heat transfer, m^2/s
ϵ_M = eddy diffusivity for momentum transfer, m^2/s
$\phi = (\Delta P/L)_{TP}/(\Delta P/L)_L$ = Lockhart-Martinelli parameter, dimensionless
μ = viscosity, N s/m^2
ν = kinematic viscosity, m^2/s
ρ = density, kg/m^3
τ = shear stress, N/m^2

Subscripts

G = the gas phase
i = the gas-liquid interface
L = the liquid phase
L, S = a superficial value in the liquid phase
w = the wall
TP = a two-phase flow value

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